Sound

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Useful Formulae

1. $v_{snd} = velocity\ of\ sound = 331\ m/s\ +\ T(°C)$
2. relation between velocity of wave (vibrating string) and of tension T in string of density μ: $v = \sqrt{T/\mu}$
3. $\Delta P = -B\frac{dD}{dx}$ Pressure related to displacement and bulk modulus

1 A Longitudinal Wave Formed by Rarifactions in the Air

Sound is a longitudinal wave that exists only in some sort of medium. A *sound of a pure frequency displaces particles in a periodic way to create expansions and compressions in the density of particles.* (Without a medium, there would be no particles. No compressions and expansions. Hence, no sound waves.) The expansions and compressions are in the same direction as the sound wave (hence, sound is a *longitudinal wave*); in fact, the expansions and compressions form the sound wave.

One representation of sound is via a *displacement wave*, where the crests and troughs of the wave are formed by the compressions and expansions of the particle density, respectively. The displacement wave representation of sound is a traveling wave going in the same direction as the oscillating particles in the medium (a.k.a., longitudinal wave): $D(x,t) = D_m sin(kx - \omega t)$, where D_m is the displacement amplitude, and the other variables have their usual meanings.

The *pressure wave* is also an equivalent representation of a sound wave. The pressure wave is related to the displacement wave via the Bulk Modulus,

$$B = -V\frac{\Delta P}{\Delta V} \tag{1}$$

$$= +A(\Delta x)\frac{\Delta P}{A(\Delta D)} \tag{2}$$

$$B = +\Delta x\frac{\Delta P}{\Delta D} \tag{3}$$

$$\Delta P = -B\frac{\Delta D}{\Delta x} \quad (4)$$

$$\Delta P = -B\frac{dD}{dx}. \quad (5)$$

A human can hear sound with frequency between 20 Hz to $20,000$ Hz.

Sound can be produced by vibrating strings. The velocity of the wave on the string relates the frequency and wavelength. The relation between velocity of wave (vibrating string) and of tension T in string of density μ: $v = \sqrt{T/\mu}$.

1.1 Sound From Tubes

Sound can also be produced by open, closed tubes, and open-closed tubes.

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In an open tube, there are antinodes at both ends. The largest wavelength λ that will fit in there is $\lambda/2$. The wave must meet the condition at both ends of the tube — the wave must have antinodes at both ends. The next largest wavelength that will fit is λ, and the next $3\lambda/2$. Thus, the general relation is: $\frac{n\lambda}{2} = L$, where L is the length of the tube and n ranges in integers.

In a closed tube, there are nodes at both ends. Sound waves must meet the condition at both ends, thus the largest wavelength λ that will fit in is, again, $\lambda/2$. (Depending on the intensity of the sound and the material of the tube, the wave might escape the boundary of the tube ends, and one might be able to hear it outside. In this case, there would only be an approximate node at the ends.)

In a tube with one side open and the other closed, there is an antinode and a node, respectively, at each end. The analysis is exactly like the two cases above, but one must count differently. The largest wavelength λ that will fit in is only $\lambda/4$. Because of the condition at the endpoints, viz., node and antinode, only odd multiples of the fundamental wavelength will fit. Thus, the general relation is: $\frac{m\lambda}{4} = L$, where m ranges in the odd integers.

2 Interference in Space

If you have two or more sources of sound seperated in space (assume same frequency and amplitude), they will interfere either *constructively* (*amplify* amplitudes) or *destructively* (*destroy* amplitudes). Destructive interference occurs when the wavelengths from the sources are, at a particular point, off by $\lambda/2$. Constructive interference occurs when the wavelengths are integer multiples of each other. (This is assuming the waves are in phase. The conditions are reversed if they are completely out of phase.)

When there are two sources of sound, interference in space amounts to a standing wave, where the sum of the wave from source 1 and the wave from souce 2 adds to form the interference wave,

$$
\begin{aligned}
D(t) &= D_1 + D_2 && (6)\\
&= D_m\left(\sin(kx-\omega t)+\sin(kx+\omega t)\right) && (7)\\
&= 2D_m \sin(kx)\cos(\omega t) && (8)
\end{aligned}
$$

3 Interference in Time (Beats)

If you have two (or more) sources of sound whose frequencies vary slightly, then the interference in time will form beats,

$$
\begin{aligned}
D(t) &= D_1 + D_2 && (9)\\
&= D_m\left(\sin(kx_0-\omega_1 t)+\sin(kx_0-\omega_2 t)\right) && (10)\\
&= 2D_m\left(\cos((\omega_1-\omega_2)t)\sin(kx_0-(\omega_1+\omega_2)t)\right) && (11)
\end{aligned}
$$

Beats occur when constructive (destructive) interference occurs. This happens at the maximum of the last line above. Thus, the frequency of the beats can be deduced by: $2\pi = \omega_1 t - \omega_2 t = 2\pi(f_1 - f_2)t \Rightarrow f_{beats} = \frac{1}{t} = f_1 - f_2 = \Delta f$

4 Sonic Booms

When an object travels greater than the speed of sound, it forms a triangular wave envelope whose angle is related to the Mach number.

5 Doppler Effect

The *Doppler Effect* occurs when either an *observer* (a.k.a., the detector) or a sound *source moves away or towards each other.*

5.1 Source Moves

If a source of sound of frequency f_s moves, then the wavelength it emits will change. If it moves with a velocity v_s towards (away from) the observer, then its wavelength will be $\lambda_{source\ emits} = \frac{v_{snd}-v_s}{f_s}$ ($\lambda_{source\ emits} = \frac{v_{snd}+v_s}{f_s}$), where v_{snd} is the velocity of sound in the medium. To get the signs right, note the following: When the source moves towards an object, the wavelength between crests will get smaller, thus the negative sign; when the source moves away from an object, the wavelength between crests will get bigger, hence the positive sign.

The frequency the observer detects is $f_{observer\ detects} = \frac{v_{snd}}{\lambda_{source\ emits}} = \frac{v_{snd}}{v_{snd} \pm v_s} f_s$, where the plus sign refers to the source moving away. To check the signs, note this: the frequency of a source approaching an observer must be bigger than that of a source receding.

5.2 Observer moves

The source of frequency f_s is at rest, but the observer is moving at velocity v_o.

The wavelength emitted by the source is $\lambda_{source\ emits} = \frac{v_{snd}}{f_s}$.

The frequency detected by the observer is $f_{observer\ detects} = \frac{v_{snd} \pm v_o}{\lambda_{source\ emits}} = \frac{v_{snd} \pm v_o}{v_{snd}} f_s$ The plus sign goes with the observer moving closer to the source, while the negative sign goes with the observer moving away from the source. (Again, when objects approach each other, the frequency heard by either object will be greater.)

5.3 Souce Moves; Wave Reflects Off Wall; Source acts as observer

The source emits a frequency f_s and moves at velocity v_s towards the observer moving at velocity v_o away from the source.

The source emits a wavelength of $\lambda_{source\ emits} = \frac{v_{snd} - v_s}{f_s}$.

The observer detects a frequency of $f = \frac{v_{snd} - v_o}{\lambda_{source\ emits}} = \frac{v_{snd} - v_o}{v_{snd} - v_s} f_s$.

The observer *reflects off this wave from the source and thus emits* a wavelength of $\lambda_{observer\ emits} = \frac{v_{snd} + v_o}{f} = \frac{v_{snd} + v_o}{v_{snd} - v_o} (v_{snd} - v_s) f_s$.

The source detects a frequency of $f = \frac{v_{snd} - v_s}{\lambda observer\ emits} = \frac{v_{snd} - v_s}{v_{snd} + v_o} \frac{v_{snd} - v_o}{v_{snd} - v_s} f_s$

5.4 A Wind Disturbs the Air

If a wind of velocity v_w adds to the speed of sound, then the speed of the medium would be $v_{medium} = v_{snd} + v_w$. Mutatis mutandis.

5.4.1 Example: Wind of velocity v_w blows, but Source and Observer Both at Rest:

Suppose the source (f_s) is located north of the observer and the wind is blowing from the north. $v_{medium} = v_{snd} + v_w$

The source emits a wavelength of $\lambda_{source\ emits} = \frac{v_{medium}}{f_s}$.

The observer hears a frequency of $f = \frac{v_{medium}}{\lambda_{source\ emits}} = f_s$.

Note the difference between a moving medium and a moving source/observer.

5.4.2 Example: EVERYTHING MOVES!

5.4.3 Example: Observer Runs Away Diagonally from the Source; Wind velocity at 45 degree angle: