AC Circuits

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An AC circuit is characterized by an *alternating current*, which is created from an *alternating voltage* source. Thus, one can formulate the basic equations of AC circuits via using either current or voltage. Here, an alternating current (i.e., oscillating in time) will be used,

$$I(t) = I_0 \sin(\omega t) \tag{1}$$

where I_0 refers to the peak current or the amplitude of the current sinusoid.

The average current is 0, since $\langle I(t) \rangle = \frac{1}{2\pi} \int_0^{2\pi} \sin(\omega t) d(\omega t) = 0$. The average current squared is $\langle I^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} I_0^2 (\sin(\omega t))^2 d(\omega t) = \frac{I_0^2}{2}$. Thus, the root mean square current is $I_{rms} = \sqrt{\langle I_0^2 \rangle} = \frac{I_0}{\sqrt{2}}$

1 R Only

Plug in (1) into Ohm's Law and you get $V = IR = I_0 R \sin(\omega t) = V_0 \sin(\omega t)$. Thus, for a resistor, the current and voltage are proportional to each other; in other words, the current and voltage are *in phase*.

$$\boxed{V(t) = V_0 \sin(\omega t) \text{ and } I(t) = I_0 \sin(\omega t)}$$

Energy flows from the source to be dissipated by the resistor. The average power (energy dissipated) is $\langle P \rangle = \langle I(t) V(t) \rangle = I_0 V_0 \langle \sin^2(\omega t) \rangle = \frac{I_0 V_0}{2} = \frac{I_0^2 R}{2} = I_{rms}^2 R = \frac{V_{rms}^2}{2}$

2 L Only

Plugging in the time derivative of (1) into $V = L\dot{I}$, one gets $V(t) = L\omega I_0 \cos(\omega t) = V_0 \sin(\omega t + \frac{\pi}{2})$. One sees that the current lags behind the voltage by $\pi/2$.

$$\boxed{V(t) = V_0 \cos(\omega t) \text{ and } I(t) = I_0 \sin(\omega t)}$$

In the above, the following relation was used: $V_0 = I_0 \omega L$. In the spirit of the familiar Ohm's Law, let's define a quantity with the same units of resistance. But, let's call it the *inductive reactance* or impedance.

$$\boxed{V_0 = IX_{L0} \Rightarrow X_{L0} = \omega L}$$

The average power is 0 because the integral of the product of $I(t)$ and $V(t)$ averages to 0. This means that energy is sent from the voltage source to the magnetic field of the inductor and back to the voltage source–with no net dissipation.

3 C Only

The voltage across a capacitor is defined as $Q = CV \Rightarrow V = \frac{Q}{C}$. But, $I = \frac{dQ}{dt} \Rightarrow Q = \int I_0 \sin(\omega t) dt = -\frac{I_0}{\omega} \cos(\omega t)$, where (1) is used for current. Thus, $V(t) = \frac{Q(t)}{C} = \frac{-I_0 \cos(\omega t)}{\omega C} = -V_0 \sin(\omega t - \frac{\pi}{2})$. The current leads the voltage by $\pi/2$.

$$\boxed{V(t) = -V_0 \cos(\omega t) \text{ and } I(t) = I_0 \sin(\omega t)}$$

In the above, the relation $V_0 = I_0 \frac{1}{\omega C}$ is used. This suggests that a *capacitative reactance* be defined as $X_C = \frac{1}{\omega C}$.

$$\boxed{V_0 = IX_{C0} \Rightarrow X_{C0} = \frac{1}{\omega C}}$$

The average power is 0 because the integral of the product of $I(t)$ and $V(t)$ averages to 0. This means that energy is sent from the voltage source to the electric field between the plates of the capacitor and back to the voltage source–with no net dissipation.

It turns out that only resistors dissipate energy in AC circuits.

4 LRC Circuit Analysis via Phasors

If you get vectors, you get phasors. The only difference is that phasors, as used here, involves rotating vectors that are either (anti)parallel or at right angles to each other.